

# **Saving Half Through Girder Bridges using Non-Linear Finite Element Analysis**

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## **ABSTRACT**

In the UK there are a large number of half through girder bridges dating from the latter part of the 19<sup>th</sup> Century that continue to provide a vital part of the transport infrastructure. The main edge girders of these bridges are frequently found to fail strength assessments even though there is no evidence of any impending failure. Their strength, according to codes of practice, relies on U-frame behaviour and a moment connection with the deck. However, in many cases it is difficult to quantify the connection due to a lack of detailed information, difficult inspection conditions and uncertainty regarding deterioration. Consequential strength assessment conservatism leads to a low connection moment capacity and, therefore, little lateral restraint to the top flange and an almost inevitable strength assessment failure.

This paper describes a new way of considering girder restraint that is not reliant on U-frame moment connections but instead utilises more definite horizontal lateral restraint provided by the in-plane shear stiffness of the deck. Current assessment rules, that consider effective length and slenderness of parts in compression, cannot be easily used to quantify it. Instead numerical simulation of girders based on non-linear finite element analysis has been used to comprehensively model failure and predict strength. Gifford have now used this technique to model 30 half through girder bridges, significantly raising strength assessment ratings and, therefore, negating costly strengthening work. This paper describes the methodology and presents a typical case study.

**KEYWORDS** Strength Assessment of Girder Bridges, Non-linear Structural Analysis

## **INTRODUCTION**

With the introduction of EU 40/44 tonne live loading all principal bridges in the UK have been assessed for strength as part of a staged assessment programme. This process has highlighted a number of common potential problems one of which is the apparent weakness of some half through deck girder bridges.

Half through deck girder bridges consist of edge girders, generally resting directly on masonry abutments, and with cross girders positioned close to the bottom flanges carrying the deck. Sometimes referred to as U-frame bridges, this arrangement is particularly efficient where headroom limits construction depth. Often over 100 years old, these bridges are generally built using riveted wrought iron or steel plates. Figure 1 shows a sketch and photograph of a typical connection between cross and edge girders in a half through girder bridge deck.

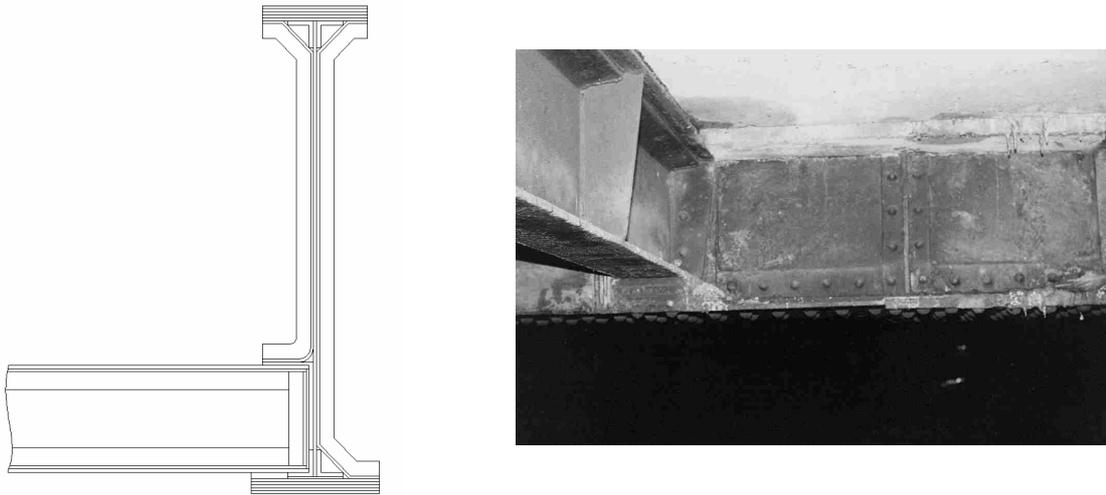


Figure 1 Typical Connection Between Cross Deck and Edge Girder

The strength of the edge girders depends on the degree of compression flange lateral stability that can be provided by a structural system analogous to a U-frame. In these bridges the U-frame is formed by the webs and stiffeners of the two edge girders and the deck cross girders. This type of structural behaviour is described in more detail by Jeffers[1,2]. In modern design[3] the stiffness and strength of the stiffeners together with the strength of the cross girder connection are sized to provide the necessary U-frame action. In assessment the connections are often insufficient to comply with design rules, or cannot be established, and this leads to a low assessed capacity for the edge girder.

### **BRIDGE ASSESSMENT**

In the UK half through girder bridges are assessed using BD21[4], which defines the overriding philosophy, generalised material characteristics and loading, and BD56[5] for rules on structural analysis and strength calculations. Although BD56 relaxes certain criteria compared to current design practice many bridges fail their strength assessments chiefly for the following reasons.

- i) It is often not practical or possible to ascertain the condition and effectiveness of the connection between the cross deck and edge girders and, therefore, U-frame behaviour cannot be relied upon.
- ii) The strength and stiffness requirements for the stiffeners in the edge girders are generally not met.
- iii) Corrosion deterioration of the connection components, stiffeners and girders is such that they cannot be depended on to carry the required loads.

Numerical modelling using the Finite Element (FE) method has been used after establishing that U-frame action cannot be relied upon. It is the next step that should be considered in attempting to boost strength assessment. Using this technique it has been possible to accurately model structural behaviour and identify so called hidden strength that does not depend on U-frame action. Two solution types were considered as follows:

## LINEAR BUCKLING ANALYSIS

This type of solution involves eigenvalue extraction and generally provides an upper bound estimate of flexural buckling for a given load. Also known as bifurcation analysis, the shape of latent buckling together with corresponding stresses can be calculated. BD56 is then used to take account of material and construction effects, to determine the slenderness parameter for lateral torsional buckling  $\lambda_{LT}$  which relates to the effective length and to calculate the ultimate compressive stress and hence the girder's capacity.

However further calculations to code requirements are then needed to check the shear capacity of the web, the connection between the cross girders and the edge girder and the adequacy of the intermediate and support stiffeners. Often these checks are time consuming and could lead to inadequate capacity, which can then only be resolved with a full geometric and material non-linear analysis. Because of these limitations and complications more holistic non-linear strength analysis is preferred for strength assessment calculations.

## NON-LINEAR STRENGTH ANALYSIS

This approach involves non-linear solutions where geometric, material, lift-off effects and load sequencing can be included and a procedure has been developed to allow for the influence of imperfections. Although computationally intensive, relying on automated iterative solutions, a comprehensive and accurate prediction of structural behaviour is possible which gives directly the ultimate strength of the edge girders. BD56 permits the use of non-linear analysis where it is necessary to more accurately predict structural behaviour.

In applying any non-linear FE analysis it is essential to validate generic results preferably by comparisons with tests or alternatively by comparing the results of similar analyses with known closed form solutions. Since FE results are not derived through established code rules other factors such as geometric imperfection, resulting from manufacturing tolerance or accidental damage, also need consideration.

## VERIFICATION

Since no test data is available, closed form solutions for a similar problem albeit greatly simplified has been used to generically verify the non-linear FE strength analysis of edge girders. Verification has been based on the buckling of a simply supported beam with a prismatic cross-section, see Figure 2, and subject to constant moment. Lateral fixity of the beam is provided at its ends. A solution for elastic buckling is given by Chatterjee[6] and repeated below.

$$M_{cr} = \frac{\pi}{L_e} \sqrt{\frac{EI_y GJ}{\alpha}} \sqrt{1 + \frac{\pi^2 EI_w}{L_e^2 GJ}}$$

For equal flange I-beams with equal area flanges

$$M_{cr} = \frac{\pi^2 EI_y}{2L_e^2} h \frac{\beta}{\sqrt{\alpha}}$$

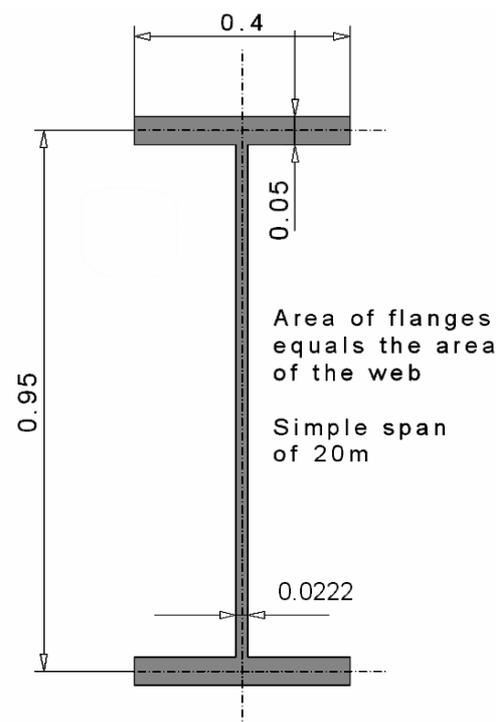


Figure 2 Beam Cross-section

$$\beta = \sqrt{1 + \frac{1}{20} \left( \frac{L_e T}{r_y D} \right)^2} \text{ where } \alpha \approx 1.0$$

$M_{cr}$  = Critical buckling moment

$L_e$  = Effective span

$I_y$  = Second moment of area about the vertical axis

$h$  = Distance between flanges centres

$T$  = Thickness of flanges

$r_y$  = Radius of gyration about the vertical axis

$D$  = Overall depth

$E$  = Elastic modulus of material

$G$  = Shear Modulus

$J$  = Torsional moment of inertia

Using the dimensions given in Figure 2 and substituting in the above equations, the critical buckling moment  $M_{cr}$  is obtained as 3.21MNm.

Comparable FE buckling analyses were carried out using the mesh shown in Figure 3. Eight node thin shell elements were used. Both linear buckling and non-linear solutions were investigated.

### Linear Buckling Comparison

For this simple problem the results of the linear buckling analysis indicated lateral torsional buckling of the girder involving lateral movement of the top flange and a critical buckling moment of 3.10MNm. The corresponding compressive stress in the top flange was 149 N/mm<sup>2</sup> and, being much less than yield, reveals an elastic geometric buckling failure. This result compares well with the closed form solution.

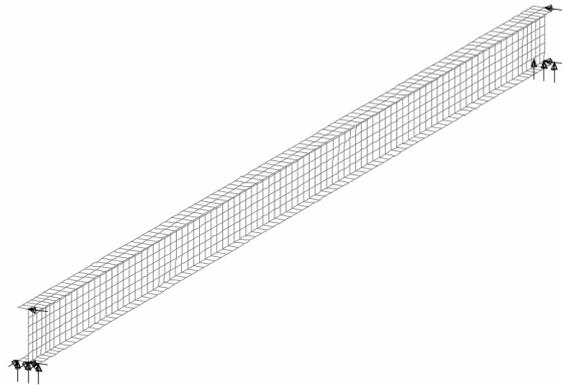


Figure 3 Verification FE Model

### Assessment Code Comparison

Applying equation 9.7.5 of BD56 leads to a slenderness parameter for lateral torsional buckling,  $\lambda_{LT}=122$ . The corresponding value derived from the code requirements is 118. This gives a design moment ( $M_D$ ) to the code of 1.98MNm compared to 1.88MNm when using  $\lambda_{LT}$  from eigenvalue analysis. Hence the eigenvalue analysis gives a design moment that is 5% less than the code. There is a significant difference between the critical buckling moment of 3.21MNm and the design moment of 1.98MNm given by the code; this is due to the allowance for imperfections,  $\gamma_m$  and  $\gamma_{\beta}$  [10], which are 1.2 and 1.1 respectively.

### Non-linear Strength Analysis Comparison

If material non-linearity is ignored and an idealised perfectly straight girder is assumed a non-linear analysis will not show any buckling and infinite strength is predicted. In keeping with reality, a meaningful result can only be obtained with this type of problem if imperfections are introduced. To investigate the influence of the imperfection on strength a range of values were modelled. Bent top flanges with mid-span offsets of 10mm, 20mm and 40mm were considered. It is worth noting that the design code[9] assumes that the top flange

imperfections are limited to 1/1000 of the span whereas for strength assessment BA56[8] advises a sensitivity approach with imperfections of 0.5 to 2 times the values used in design. Geometrically non-linear and elastic strength results for the critical buckling moment  $M_{cr}$  are included in Figure 4 and all asymptote to a value close to the linear buckling analysis. These results show that the non-linear analysis is capable of simulating the behaviour of a girder with an imperfection under bending through lateral torsional buckling to failure.

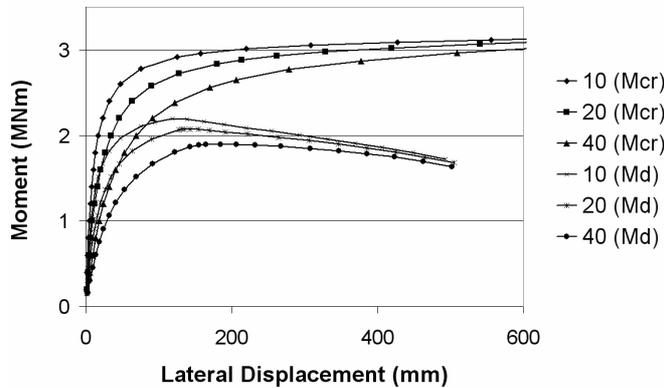


Figure 4 Verification, Influence of Imperfection (mm)

A further analysis has also been undertaken to investigate the influence of material non-linearity. The results of these analyses, showing the calculated design moment  $M_d$ , are shown in Figure 4. These results include  $\gamma_m$  and  $\gamma_{f3}$ . The calculated design moments are 2.20MNm, 2.07MNm and 1.90MNm. The imperfection code requirement design moment is 2.07MNm, which is 5% higher than the code value. This shows good agreement with the code for this simple form of girder. For

assessment purposes it would be normal to assume the larger imperfection that gives 1.90MNm, which is 4% less than the code value.

The results obtained with all types of non-linear analyses,  $M_{cr}$  and  $M_d$ , compare well with either closed form elastic solutions or assessment code calculations.

### NEW LATERAL DECK RESTRAINT

Although it is seldom possible to reliably use U-frame action to provide lateral restraint in the strength assessment of old half through girder bridges another restraint exists that is more certain but is not yet recognised either in design or assessment codes of practice. This restraint does not rely on any moment connection with the deck but simply a translational link at the level of connection to the deck, which is far easier to justify. Through this link the high in-plane stiffness of the deck acts to keep the edge girders straight at the level of the connection. Through the use of FE modelling it can be shown that this type of edge girder stiffening elevates edge girder strength and in many cases improves bridge assessment ratings. A further convenience in modelling this new restraint is that multipoint constraint equations can be used to simply define the straight-line stiffener.

To illustrate the benefits of this stiffening restraint the simple flanged beam model used for verification has been modified. A line at 0.25m from the soffit of the edge girder has been restrained to represent an attached bridge deck so that along its length lateral displacement is constant. The non-linear analysis now gives a design moment  $M_d$  of 2.83MNm, which indicates an improvement of 37% over the unrestrained girder.

### TYPICAL CASE STUDY

This case study is based on a typical single span half through girder bridge on brick abutments. The bridge is shown in Figure 5 and crosses the railway at a skew of  $42^\circ$  and has a clear skew span of 10.35m. The carriageway is 4.6m wide with no verges and the date of construction is believed to be 1854. It comprises longitudinal wrought iron riveted plate

girders with tee web stiffeners, transversely connected by wrought iron girders supporting brick jack arches.

The assessment, in accordance with BD21/97[7] and BD56[5], concluded that the bridge rating was limited by the bending capacity of edge girders and was less than dead load. The transverse girders are not aligned with the web stiffeners and there is evidence of minor corrosion to the inner face of the edge girders. Hence, the connection between the transverse girders and the edge girders may have suffered corrosion and cannot be fully relied upon. Therefore U-frame action could not be taken into account in the initial assessment.



Figure 5 Typical Half Through Girder Bridge on Masonry Abutments

### Finite Element Model

One of the edge girders was modelled without any bending restraint provided by the transverse girders. However, the new restraint associated with the in-plane shear stiffness of the bridge deck was included. This constraint allowed transverse displacement, but kept the edge girder straight at the level of the transverse girders. It did not provide any direct rotational restraint to the edge girder.

The model consisted of 3D second order thin shell elements, which modelled the flanges, web panels, flange angles, tee stiffeners and endplates. Thick shell formulation was not necessary as shear deformation normal to the plates was insignificant. The thickness of the elements was modified to take into account any corrosion losses.

Longitudinal and lateral supports were provided to prevent horizontal rigid body motion. Vertical support was achieved through non-linear contact elements so that any lift-off at the bearings could be represented; this ensured that torsional restraint at the supports, important for calculating lateral torsion buckling, was not overestimated. The possibility of transverse load eccentricity at the bearing stiffeners and the consequential reduction in stiffener capacity was therefore incorporated in the analysis.

### Linear Buckling Analysis

Linear buckling analyses of the model was carried out in order to obtain the likely modes of failure under full Ultimate Limit State (ULS) loading at the bifurcation point and the form of the imperfection distribution used in the non-linear analyses.

Using an inverse iteration algorithm the first mode as illustrated in Figure 6 indicated that lateral torsional buckling was dominant, but also showed signs of web buckling. The lowest buckling load factor was found to be 3.98, which in this case was a factor on the applied load vector that included dead, wind and 40/44

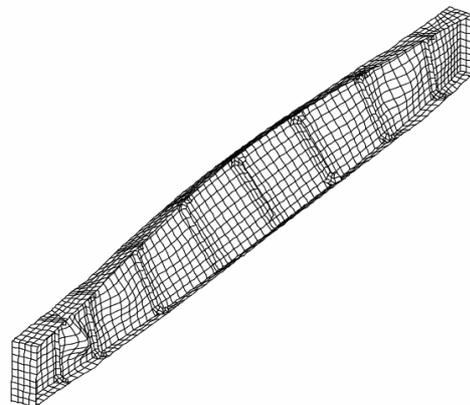


Figure 6 FE Model and Lowest Buckling Mode

tonnes Assessment Live Loading at ULS. Not surprisingly, the linear buckling analysis predicted the onset of web buckling and lateral torsional buckling at loads greater than the required Assessment Live Loading.

### Non-Linear Analysis

A yield stress of  $220\text{N/mm}^2$  was used for wrought iron[7] and further adjustments were made for rivet holes in the tension flange by modifying yield in the tension flange in proportion to the ratio of the net area of the flange to the gross area, where the net area was the gross area less the rivet holes. The critical live load case was found from simpler linear analyses. Solutions were obtained using an iterative Newton-Raphson load control procedure, which automatically invoked the arc length method to follow softening and post-buckling behaviour.

Imperfections in the geometry were included in the non-linear analysis by considering workmanship and the maximum acceptable values in design [9]. BA56/96[8] suggests using half and twice these values in order to assess buckling sensitivity. The approach used in this investigation was to adopt the larger values.

Two geometric imperfection cases were analysed; the first representing a bowed web panel and the second representing a laterally deformed compression flange. The resulting deflected shape and corresponding stresses for the bowed girder, which was found to be critical, are given in Figures 7. The dark grey on the contour plot highlights the areas of the girder that have yielded. The majority of the yielding occurred in the top and bottom flanges at mid-span. Localised areas of yielding occurred where deck loads were applied to the model and to the first internal stiffener at one support.

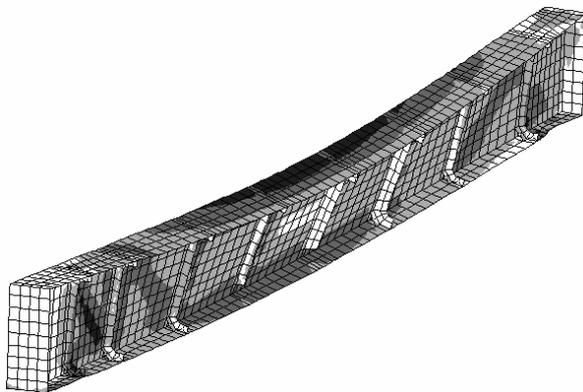


Figure 7 Contoured Von Mises Stresses

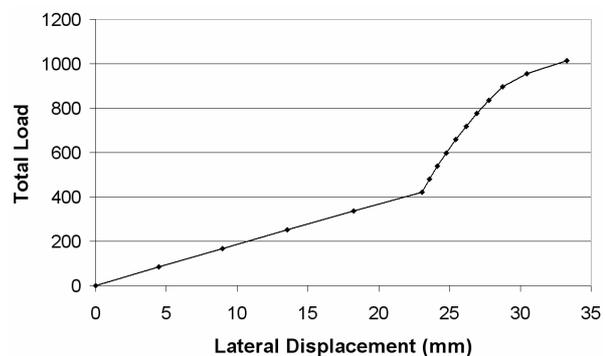


Figure 8 Lateral Deflection of Top Flange

The maximum predicted Von Mises strain was 0.73%, indicating continued ductility and within acceptable limits. The lateral deflection of the top flange at mid-span against the total vertical load is shown in Figure 8. The initial phase of loading, which included wind load, did not show any significant non-linear behaviour. However, the second phase did exhibit non-linear behaviour although deflections were small; characteristics of a girder cable of carrying loads beyond ULS. It was therefore concluded, that based on this assessment, the capacity of the edge girders was increased from less than dead load to 40/44 tonnes Assessment Live Loading.

This example illustrates the significant improvement in the assessed capacity that can be achieved with this form of non-linear analysis. For this bridge it has removed the need for costly strengthening works or bridge replacement.

## CONCLUDING REMARKS

- A new restraint has been recognised that exists in all half through deck girder bridges that can be used through non-linear finite element analysis to significantly increase bridge strength assessments. This restraint, which does not rely on moment continuity with the deck and is currently not recognised by design and assessment codes of practice, has now been applied in the assessment of 30 bridges.
- Non-linear finite element analysis is a holistic method of determining girder strength and with a suitable mesh circumvents separate code based strength calculations.
- Linear buckling solutions in conjunction with assessment code calculations can be used to predict the strength of complex edge girders. However additional calculations to code requirements are needed to check the capacity of the web for shear, the connections, the intermediate stiffeners and the support stiffeners. These checks can lead to inadequate capacity, which can then only be resolved with a full non-linear analysis.

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